Proximity Effect Physics

Going beyond tunnel junctions (SNS, microbridges, SFS, …)

These slides are based on those prepared by Prof. Dale van Harlingen (UIUC)
for his class Physics 498 Superconducting Quantum Devices

Outline:

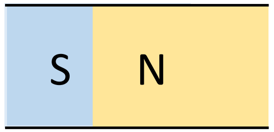
Proximity Effect Phenomenology

Screening in Proximity-Coupled SC/Normal Bilayers

Andreev Scattering

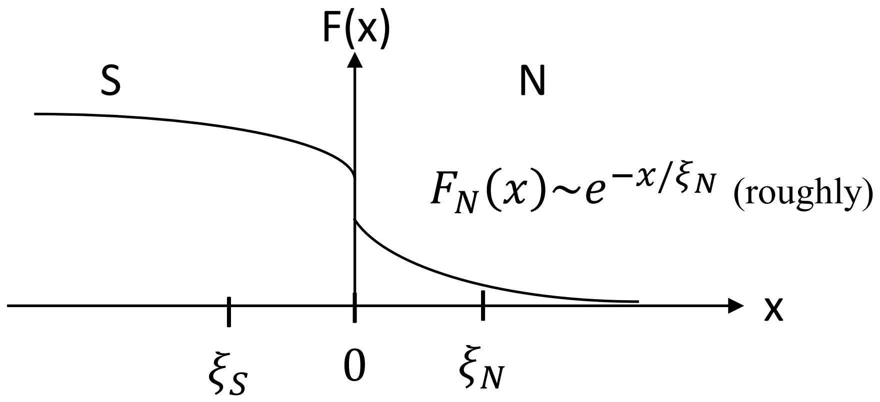
SNS Josephson junction

PROXIMITY EFFECT



Leakage of Cooper pairs out of S into N

Leakage of quasiparticles out of N into S



Results in the spatial variation of parameters

$$\Delta_k = -\sum_l V_{kl} \langle c_{-l\downarrow}\, c_{l\uparrow} \rangle \;=\; -\sum_\ell V_{k\ell}\, u_\ell^* v_\ell (1 - F_k(E))$$

$$\xi_S = \frac{\hbar v_F}{\pi \Delta} \qquad \xi_N = \frac{\hbar v_{FN}}{k_B T} \quad \text{clean}$$

$$= \sqrt{\frac{\hbar v_{FN} \ell}{6\pi k_B T}} \quad \text{dirty}$$

$$= \sqrt{\frac{\hbar v_F \ell}{6\pi k_B (T - Tc)}} \quad \text{SC}$$

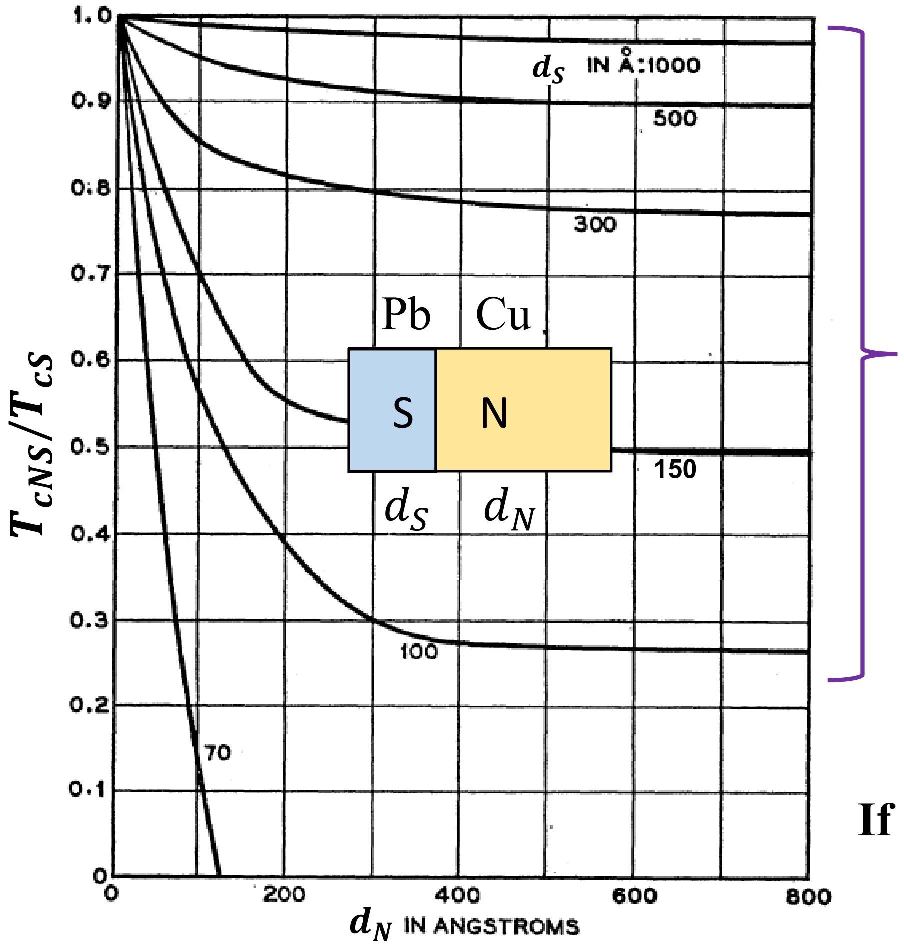
$$\Delta(\vec{r}) \;=\; g(\vec{r}) \;\; F(\vec{r})$$

Real-space generalization of BCS theory (Lecture 14)

- $\Delta(\vec{r})$: order parameter
- $g(\vec{r})$: effective pairing interaction
- $F(\vec{r})$: pair correlation function – OR – condensate amplitude

Proximity effect $\Rightarrow$ $F\,(\vec{r})$ smears out across the interface

T_c of Proximity-Coupled S/N Bilayers

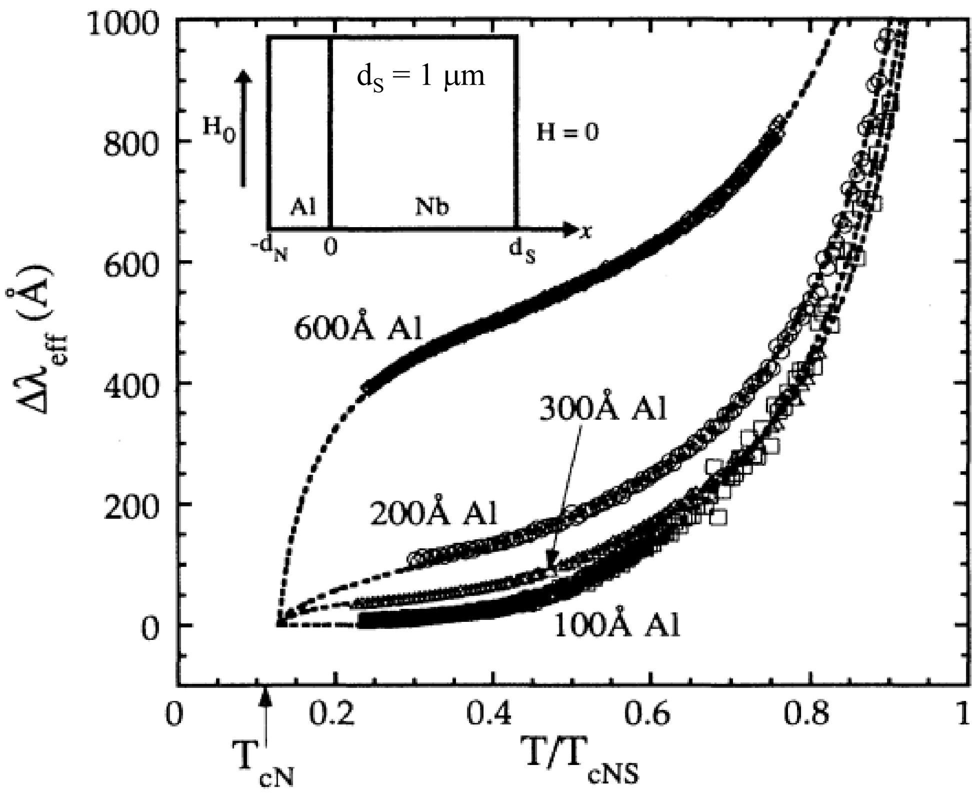


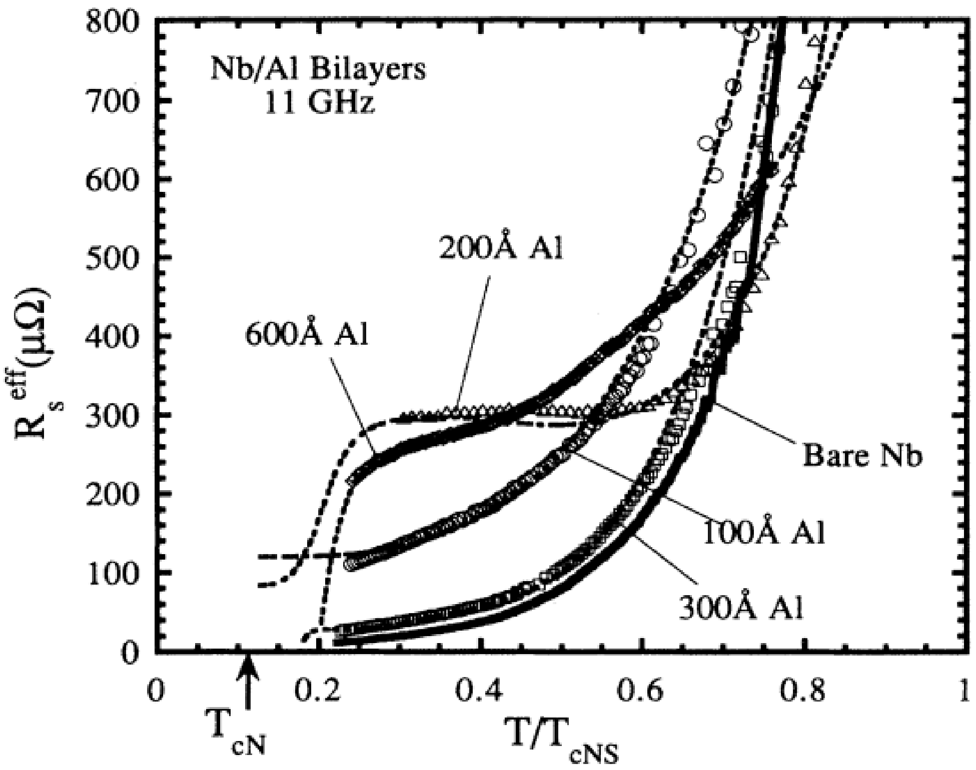
T_{cNS} saturates when $d_S > \xi_S = \frac{\hbar v_F}{\pi \Delta}$

If $d_S < \xi_S$ then $T_{cNS} \rightarrow 0$ at some d_N

N. R. Werthamer, "Theory of the Superconducting Transition Temperature and Energy Gap Function of Superposed Metal Films," Phys Rev **132** (6), 2440 (1963).

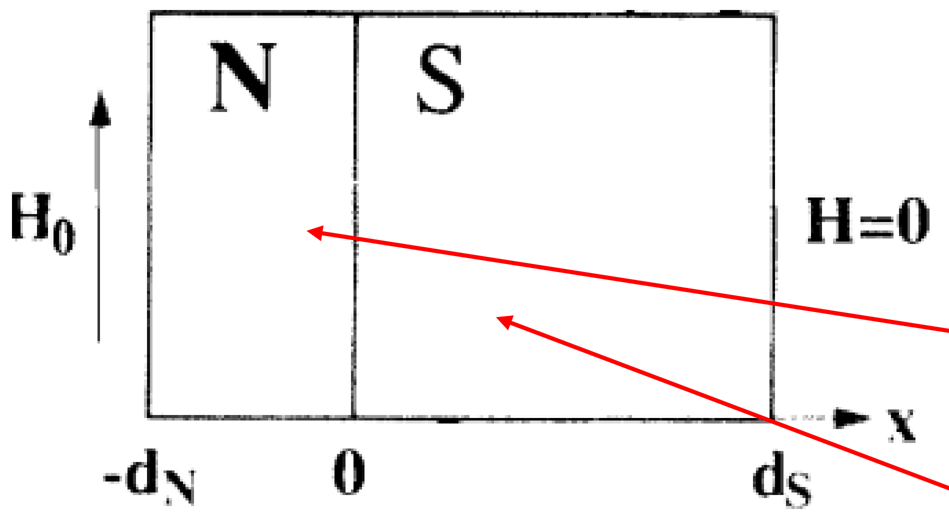
How are Electromagnetic Fields Screened by a Proximity-Coupled SC/Normal Bilayer?
S/S′ (Nb : T_c =9.2 K / Al : T_c = 1.2 K) Case





Michael S. Pambianchi, S. N. Mao, and Steven M. Anlage, "**Microwave Surface Impedance of Proximity-Coupled Nb/Al Bilayer Films**," Phys. Rev. B 52, 4477 (1995).

How are Electromagnetic Fields Screened by a Proximity-Coupled SC/Normal Bilayer?
Generalized London Model



Solve the generalized London equation for $H(x)$ in the case of position-dependent screening strength $\lambda(x)$

$$H''(x)+\frac{2}{\lambda(x)}\lambda'(x)H'(x)-\frac{1}{\lambda^2(x)}H(x)=0$$

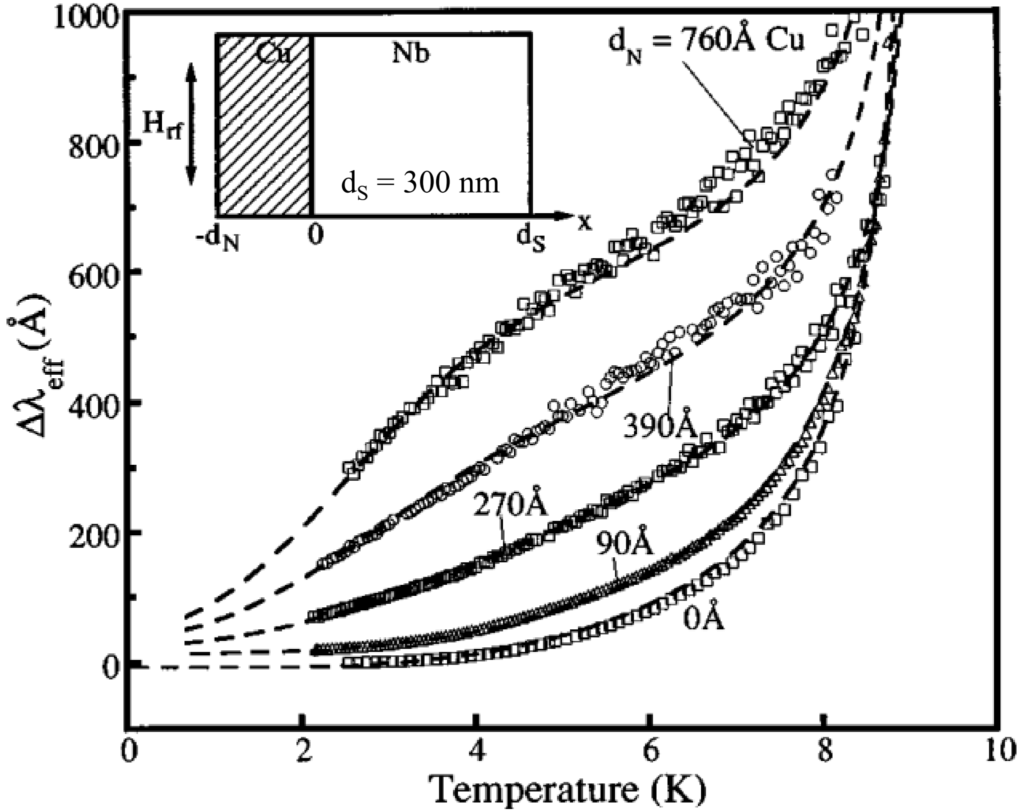
with

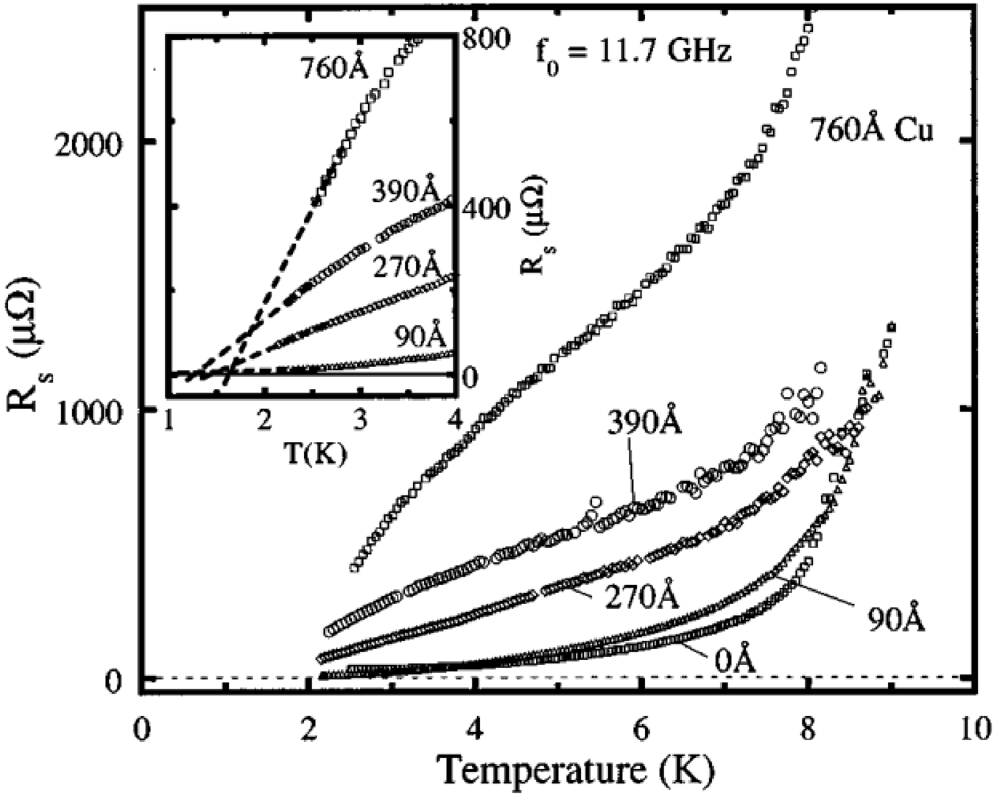
$$\lambda_N(x,T)^{-2}=\mu_0\sigma_N/(\hbar\pi k_B T)\Delta_N^2(x,T)\times\psi'[1/2-\hbar DK^2/(4\pi k_B T)]$$

$$\lambda_S(x,T)=\lambda_{S\,\mathrm{bulk}}(T)\coth[(x-x_0)/2^{1/2}\xi_S(T)]$$

Michael S. Pambianchi, S. N. Mao, and Steven M. Anlage, "**Microwave Surface Impedance of Proximity-Coupled Nb/Al Bilayer Films**," Phys. Rev. B 52, 4477 (1995).

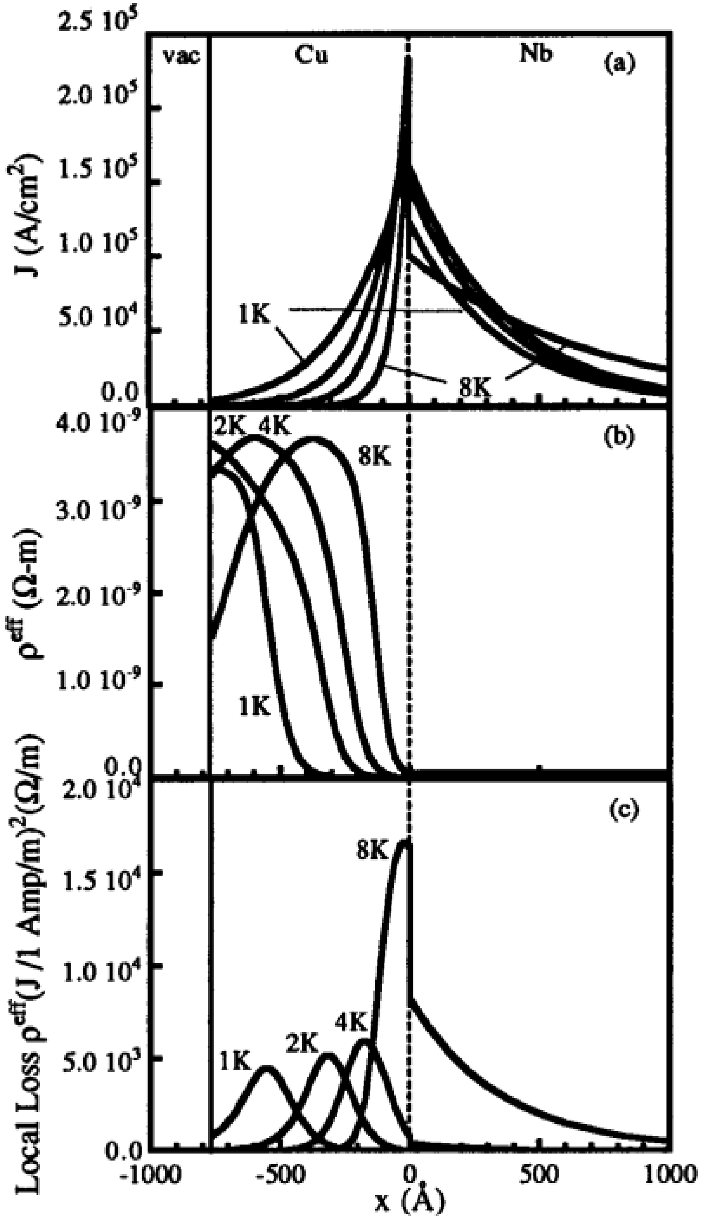
How are Electromagnetic Fields Screened by a Proximity-Coupled SC/Normal Bilayer?
S/N (Nb : T_c =9.2 K / Cu : T_c = 0) Case





M. S. Pambianchi, Lie Chen, and Steven M. Anlage, "**Complex Conductivity of Proximity-Superconducting Nb/Cu Bilayers**," Phys. Rev. B 54, 3508-3513 (1996).

How are Electromagnetic Fields Screened by a Proximity-Coupled SC/Normal Bilayer?
S/N Case



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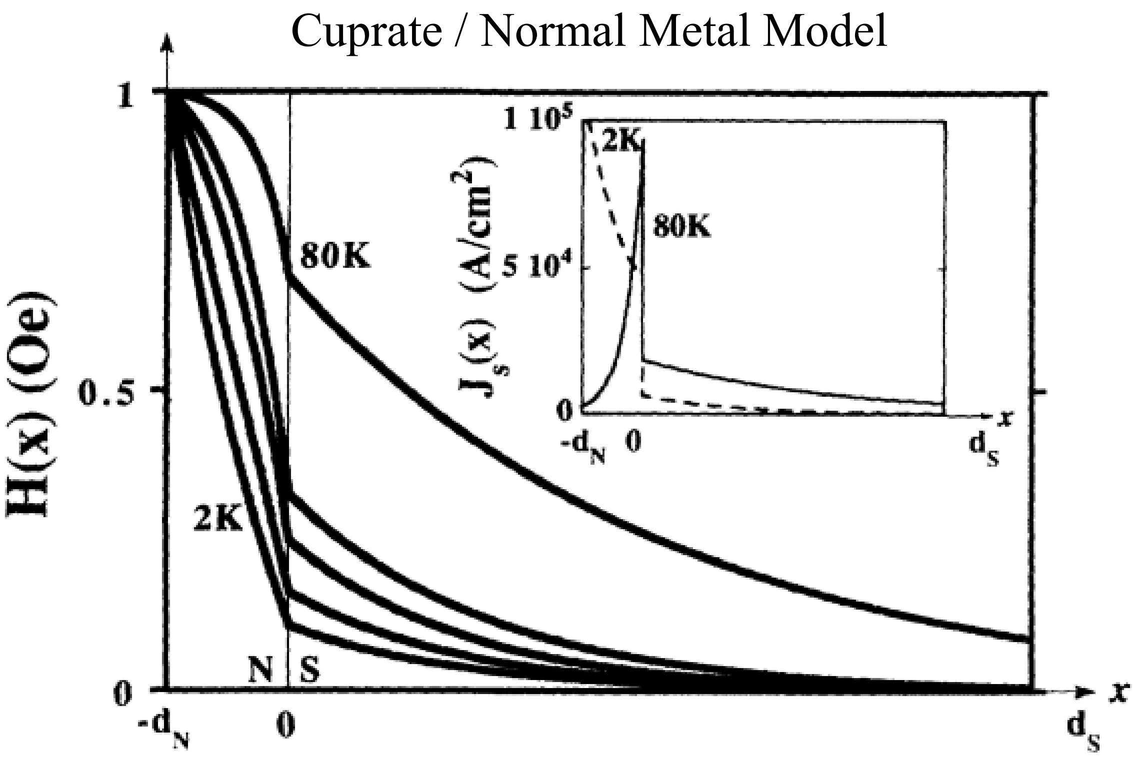


Michael S. Pambianchi, Jian Mao, and Steven M. Anlage, "**Magnetic Screening in Proximity-Coupled Superconductor/ Normal-Metal Bilayers**," Phys. Rev. B 50, 13659 (1994).

SOVIET PHYSICS JETP VOLUME 19, NUMBER 5 NOVEMBER, 1964

Andreev Scattering

THE THERMAL CONDUCTIVITY OF THE INTERMEDIATE STATE IN SUPERCONDUCTORS

A. F. ANDREEV

Institute for Physics Problems, U.S.S.R. Academy of Sciences

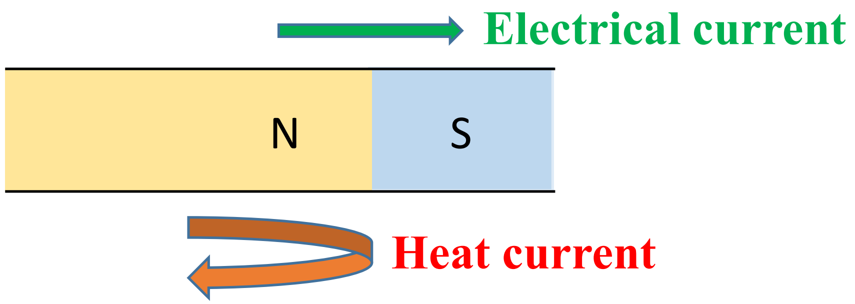
Submitted to JETP editor November 27, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) **46**, 1823-1828 (May, 1964)

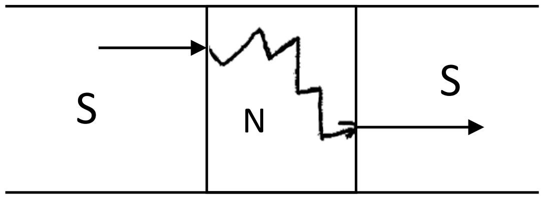
It is shown that, owing to over-the-barrier reflection of electron excitations at the boundary of the normal and superconducting phases, a temperature drop occurs when there is a flow of heat. The additional thermal resistance of a superconductor in the intermediate state is calculated. It is shown that it increases exponentially as the temperature is lowered and does not depend on the electron mean free path.

Andreev wanted to understand why, at a clean Normal/Superconductor interface, for sub-gap ($E < \Delta$) quasiparticles:

1) The electrical current passes through the interface with no trouble, while
2) Thermal current was blocked

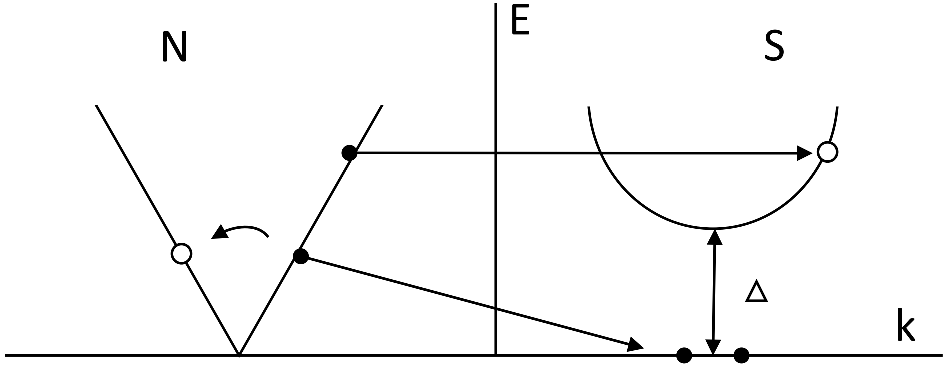


Microscopic picture



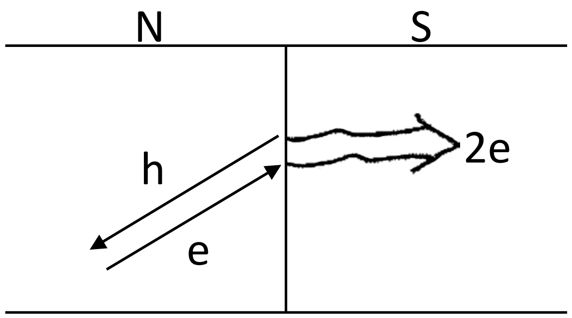
Normal conduction by diffusion through normal metal
How is phase coherence maintained?

ANDREEV REFLECTION – process for charge transport across $N \rightarrow S$ interface



electrons above the gap can find states in the superconductor

electrons below have no available states



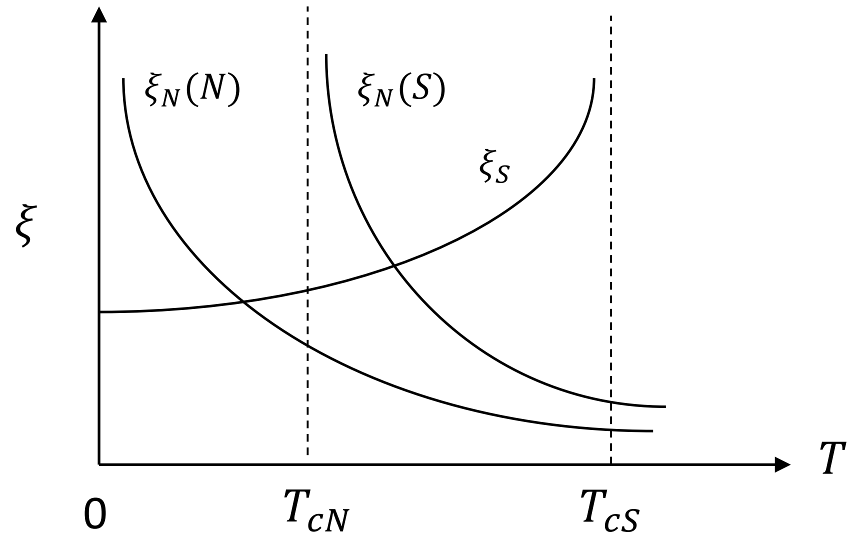
electron from N incident on the barrier

hole created in N "retroreflected" to conserve energy and momentum

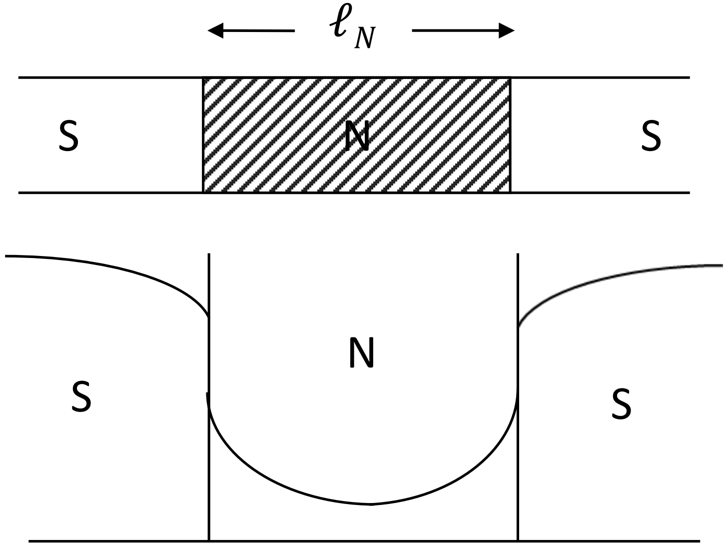
Cooper pair created in S

probability depends on E/Δ

Temperature dependence



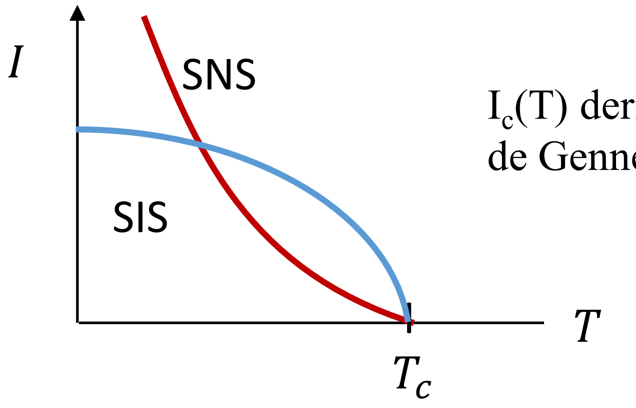
Spatial dependence



Critical current

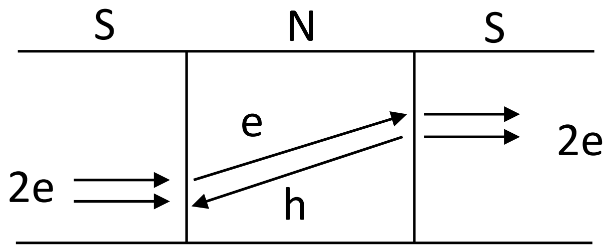
$$I_c(T) = I_{co}\left(\frac{\xi_N(T)}{\xi_S(T)}\right)^2 e^{-\ell_N/\xi_N(T)}$$

$$\approx I_{co}\left(1-\frac{T}{T_c}\right)^2 e^{-\ell_N/\xi_N(T)}$$



$I_c(T)$ derivation: see de Gennes book, p. 238

SNS Josephson junction

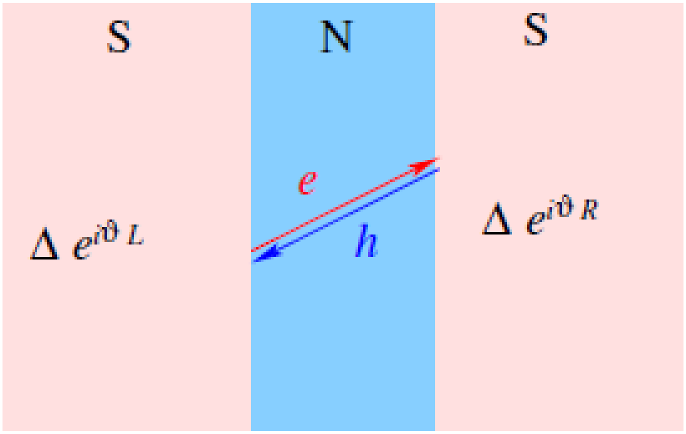


Forward and back scattered electrons and holes are coherent

(1) broad quasiparticle states in N

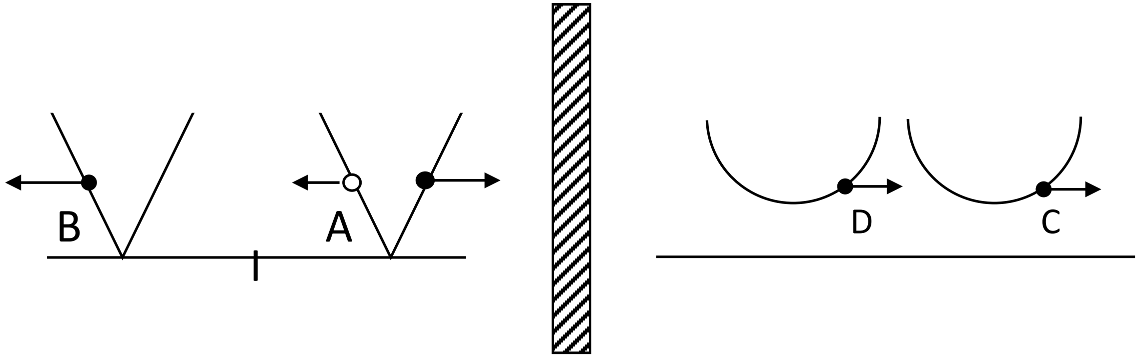
(2) maintain phase coherence across the junction

"bound states carry supercurrent"



J. A. Sauls, "Andreev bound states and their signatures," Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences **376** (2125), 20180140 (2018).

Andreev reflection – (1964)   developed to explain as a decrease in the thermal conductivity at low T



extended by   Blonder-Tinkham-Klapwijk (BTK)

Predicts shape of tunneling characteristics as the barrier height changes

Bound states   reflection & phase coherence $\Rightarrow$ interference (standing waves)

Schroedinger equations for qp's

Solve Bogoliubov-deGennes equations

$$u_k, v_k \rightarrow u(r), v(r)$$

momentum space   position space

$$H_0\, u(r) + \Delta(r)\, V(r) = E\, u(r)$$

$$H_0^*\, v(r) + \Delta^*(r)\, u(r) = E\, v(r)$$

$$\Delta(r) = g(r) F(r)$$

For $\Delta(r) = 0$, decouple into:

$$H_0 u(r) = E u(r)$$

$$-H_0^* v(r) = E v(r)$$

define normal qp's (excitations)

Josephson Effects in Weak Links

tunneling derived for SIS tunnel junction $\Rightarrow$ $I_N = G(V)V$ (qp)

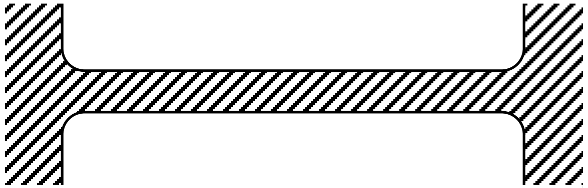
$$I_S = \frac{\pi\Delta}{2eR}$$ (pair)

Josephson effect is more universal $\Rightarrow$ QM description in terms of phase-locking of superconductor islands

How to define Josephson effect?

Not supercurrent common in superconductors

Not $I = I_c \sin\phi$



Key property: Periodic CPR $I = I(\phi)$

(Waldram) $\psi \rightarrow 0$ for some phase ϕ (usually $\phi \sim \pi$)

Key experiments: CPR measurements (not common/challenging)

AC-induced steps (steps)

SQUID behavior/diffraction pattern behavior

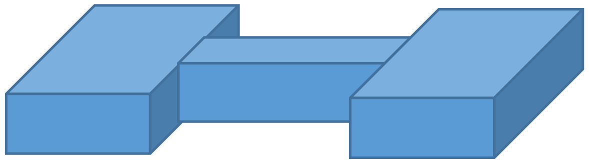
What is different ? Critical current vs. T $\rightarrow$ how excitation suppress superconductivity

Critical current vs. B $\rightarrow$ current-phase relation

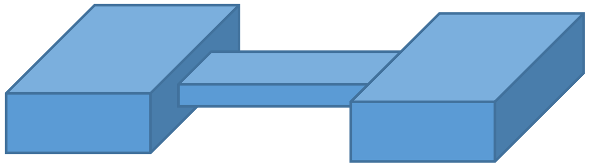
Types of Josephson junctions

S-I-S	insulator
S-N-S	normal metal
S-F-S	ferromagnet
S-Sm-S	semiconductor
S-TI-S	topological insulator
S-v-S	vacuum (STM)
S-g-S	graphene

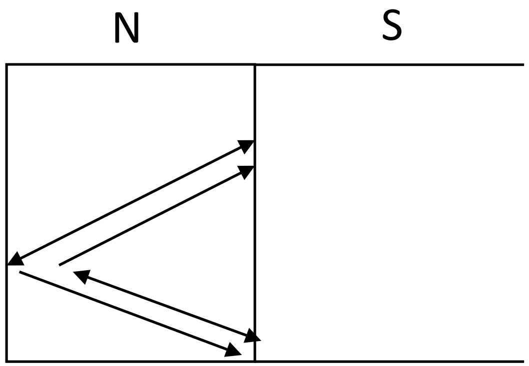
microbridge

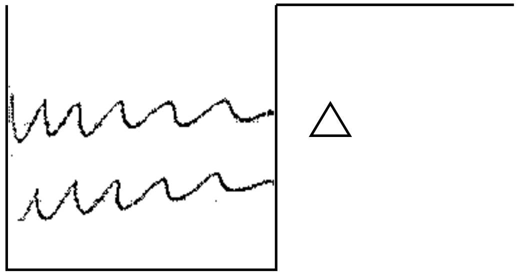


"Dayem bridge" (variable thickness microbridge)



More oscillations --- the deGennes – St. James oscillations





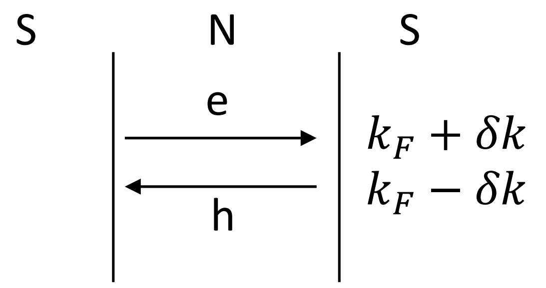
$$E_n = \frac{\pi^2 \xi \Delta}{2d}\left(u + \frac{1}{2}\right) < \Delta$$

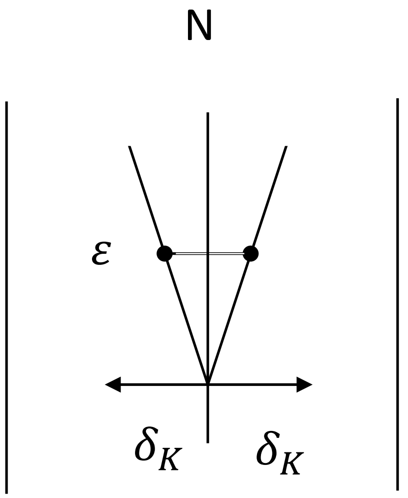
$$n \leq \frac{2d}{\pi^2 \xi}$$

Like particle in a box, except E near E_F so phase winds

δK looks like Schroedinger equation

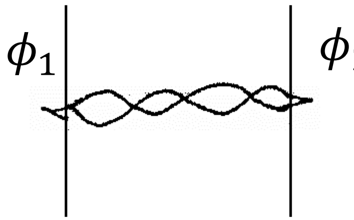
Ballistic SNS junction





$\varepsilon = \hbar v_F \delta k$ bound state energy

$$\delta k = \frac{2\pi}{\lambda} = \frac{\varepsilon}{\hbar v_F} \Rightarrow \lambda = \frac{h v_F}{\varepsilon}$$



evanescent tail

$$\phi + 2\cos^{-1}\left(\frac{\varepsilon}{\Delta}\right) + \frac{2d}{\lambda} = 2\pi h \qquad \xi = \frac{\hbar v_F}{\pi \Delta}$$

phase diff. evanescent path

$$\phi = 0 \qquad \varepsilon_n = \frac{\pi^2 \xi \Delta}{4d}\left(n + \frac{1}{2}\right) < \Delta$$

$$n < \frac{4d}{\pi^2 \xi}$$ currents cancel

ϕ finite currents do not cancel $\Rightarrow$ net supercurrent

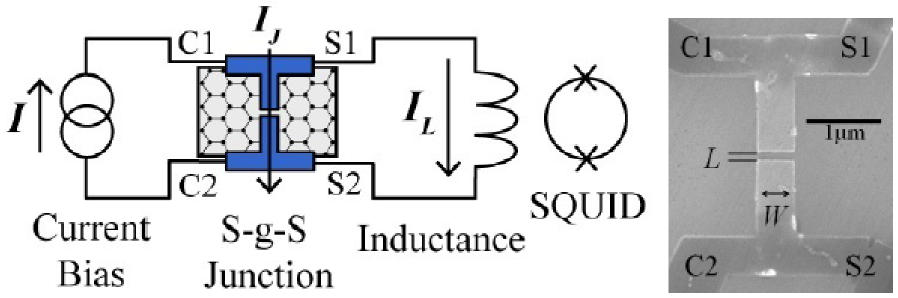
Superconductor-Graphene-Superconductor Junctions

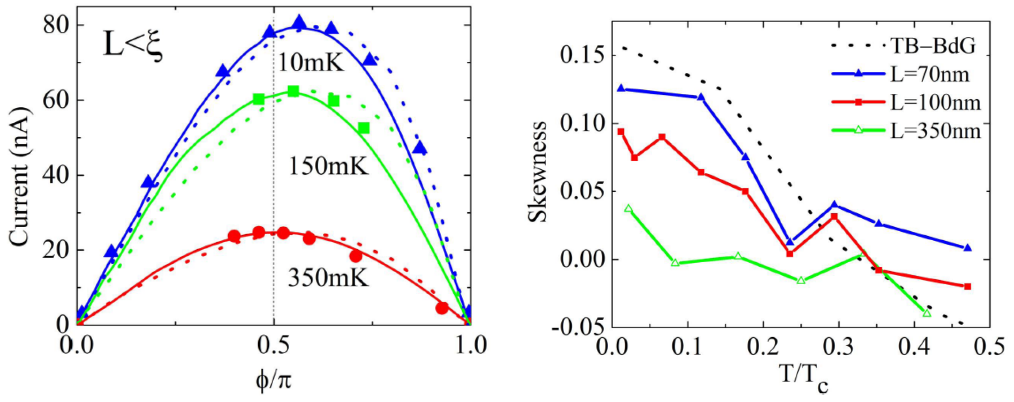
High transparency states give higher harmonic contributions to the CPR, inducing skewness

$$I_c = \frac{e\Delta}{h}\sum_{i=0}^{\infty}\frac{T_n \sin(\phi)}{\sqrt{1 - T_n \sin^2(\phi/2)}} \quad \Rightarrow \quad I_c(\phi) = \frac{e\Delta}{h}\frac{2W}{L}\cos(\phi/2)\tanh^{-1}(\sin(\phi/2))$$

Titov and Beenaker,
PRB 94, 041401 (2006)

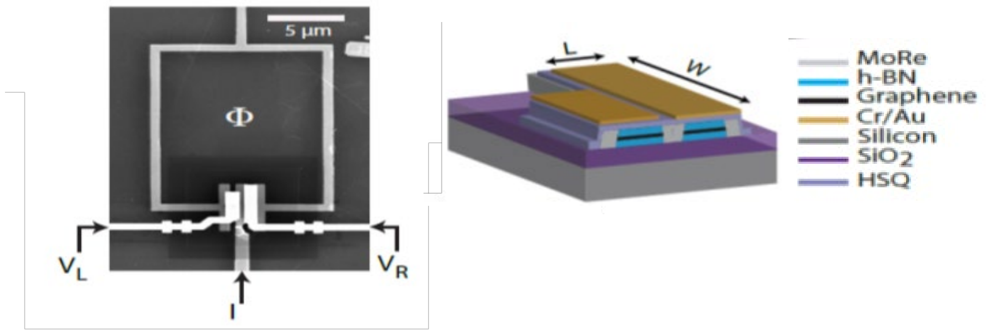
Interferometer technique (Urbana)

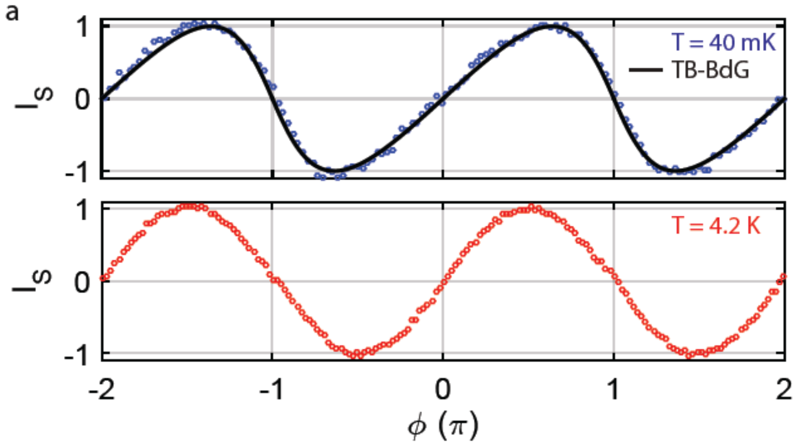




C. English et al., PRB 94, 115435 (2016)

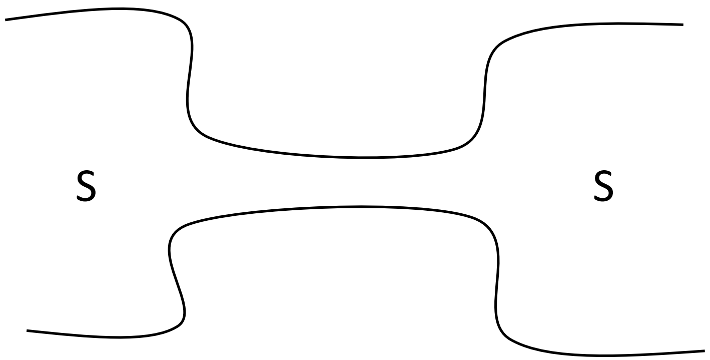
Asymmetric SQUID technique (Delft)



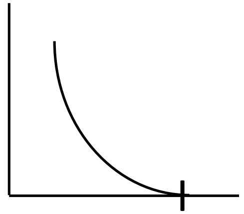


Nanda et al., arXiv:1612.06895v2

Microbridge solve GL equations



$$J_c^{GL} = \frac{c}{3\sqrt{6}\pi}\frac{H_c(T)}{\lambda(T)} \sim \left[1-\left(\frac{T}{Tc}\right)^2\right]^{3/2}$$



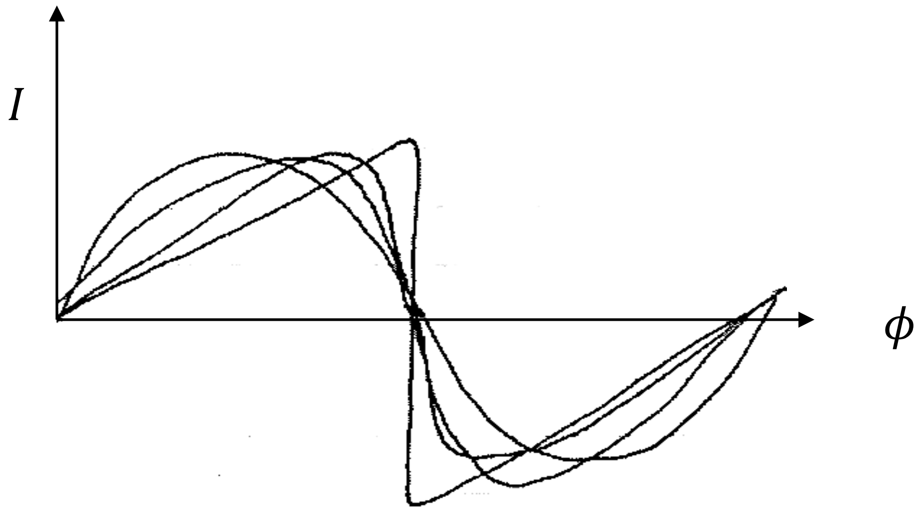
current & piling

Crossover from bulk SC behavior to Josephson behavior

↑ long wide

↑ short narrow

Modification of CPR :

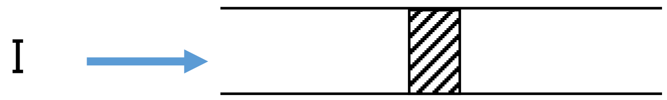


Superconducting weak links
K. K. Likharev
Rev. Mod. Phys. **51**, 101 (1979)

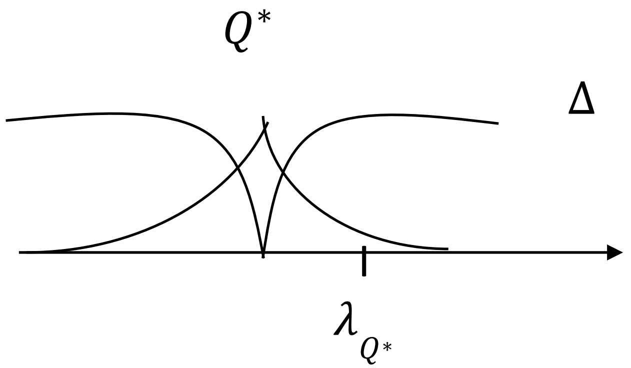
Phase slip centers:

How does a μbridge go normal?

Generates local “Phase slip center” --- like 1D vortex

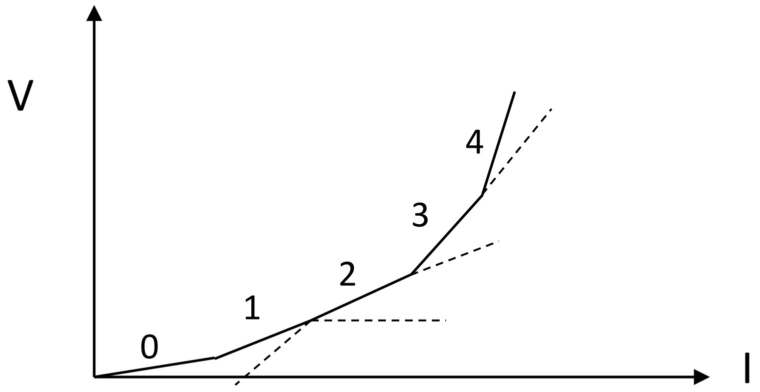


Non-equilibrium region
nucleates when Ic exceeded



Charge imbalance --- excess of charge in the qp distribution

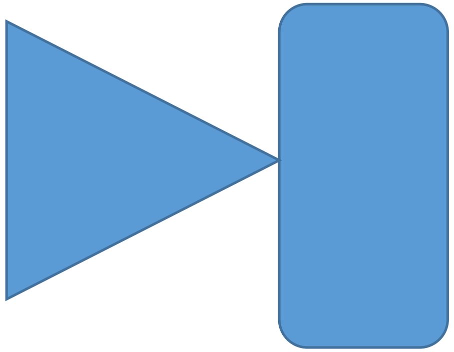
$$R_{PSC} = \frac{2\lambda_Q \rho}{A} = \left(\frac{2\lambda_Q}{L}\right) R_N$$



discrete PSC’s – spread out evenly in principle but pinned by defects in real samples

Quantum phase slips? --- still debated whether PSC can be generated by macroscopic quantum tunneling (Alexey Bezryadin has studied this)

<u>Point contact Josephson junctions</u>



SIS?

SNS?

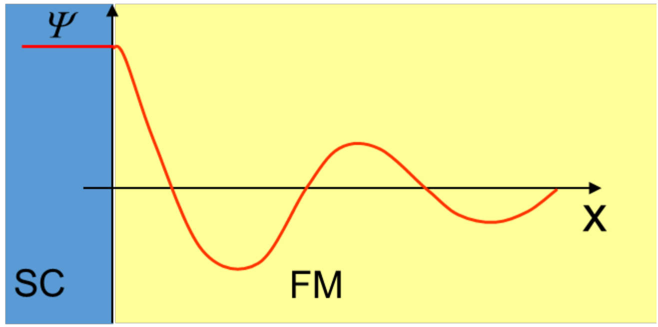
microbridge?

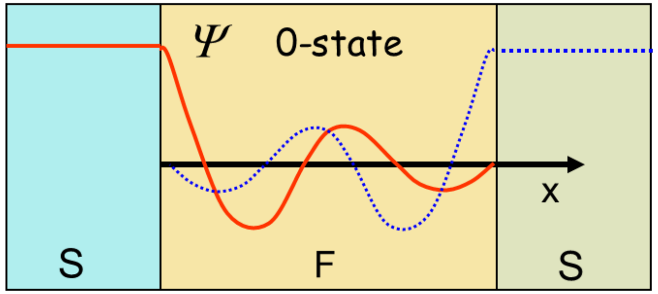
Common technique: "break junctions" --- break a wire and them re-contact

Useful to make clean contacts in exotic materials, e.g. HTSC

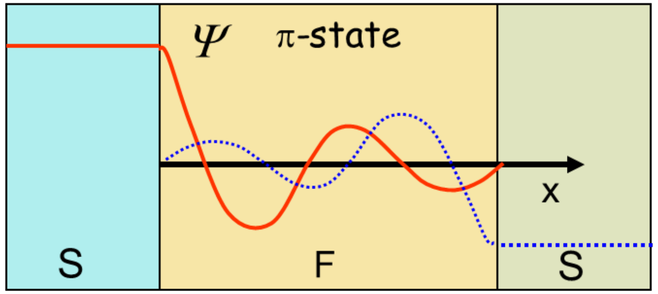
Superconductor-Ferromagnet-Superconductor Junctions

SC order parameter decays AND oscillates due to magnetism --- FFLO state





$\lambda \sim nd$



$\lambda \sim (n+\frac{1}{2})\, d$

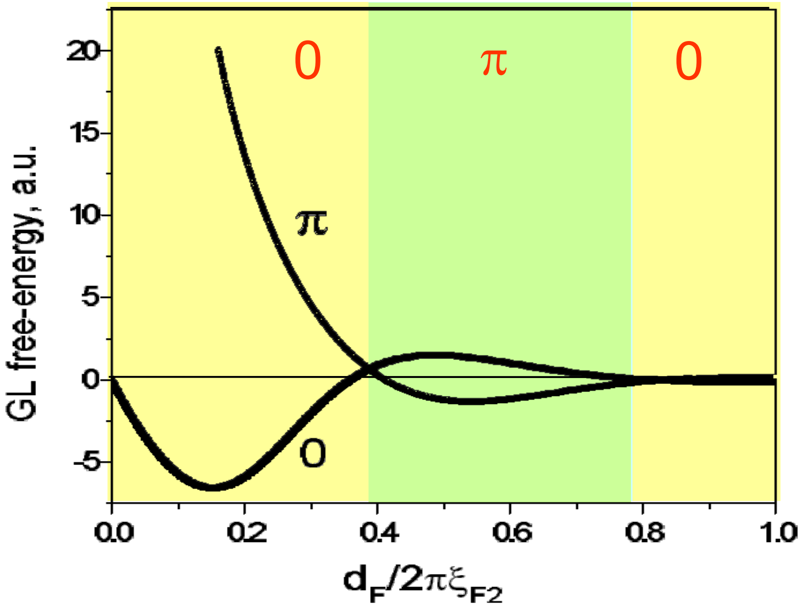
$$\Psi(x) \sim \cos\left(\frac{x}{\lambda_F}\right) \exp\left(-\frac{x}{\xi_F}\right)$$

Order parameter oscillations

Proximity decay

$$\lambda_F = \xi_F = \left(\frac{\hbar D}{2(\pi k_B T + iE_{ex})}\right)^{1/2}$$

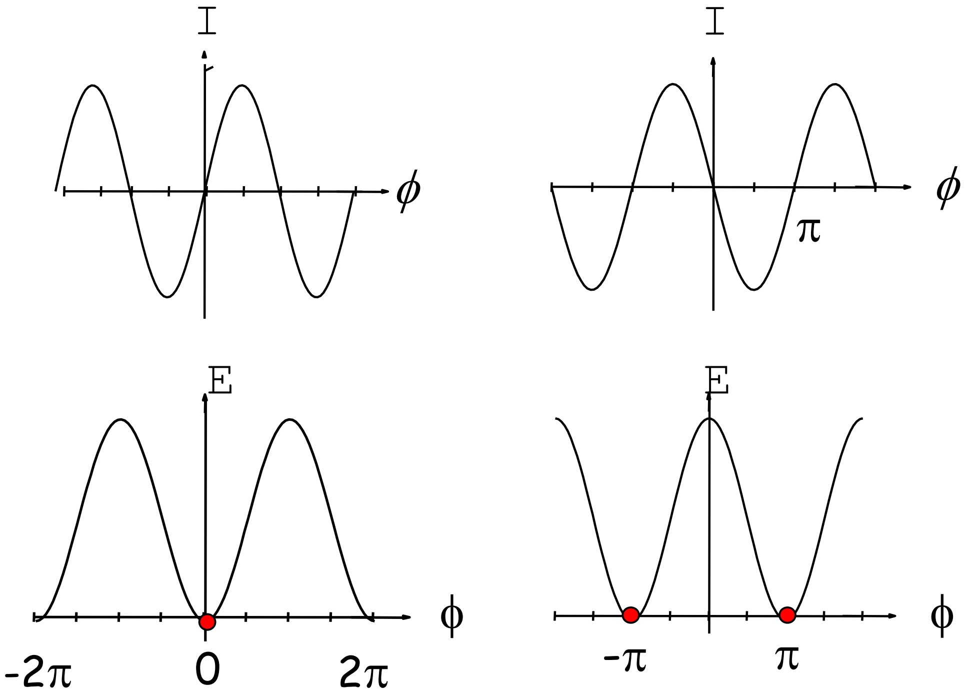
Can move across the 0-π transition by changing the barrier thickness or the temperature



π-Josephson junction

0-junction — minimum energy at 0

π-junction — minimum energy at π

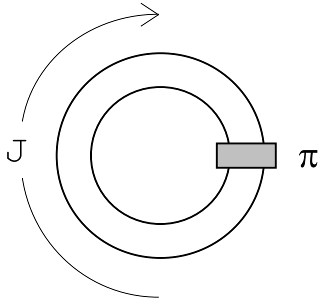


$$I = I_c \sin(\pi+\phi) = -I_c \sin\phi$$

negative critical current

$$E = E_J\,[1 - \cos(\pi+\phi)]$$

$$= E_J\,[1 + \cos\phi]$$

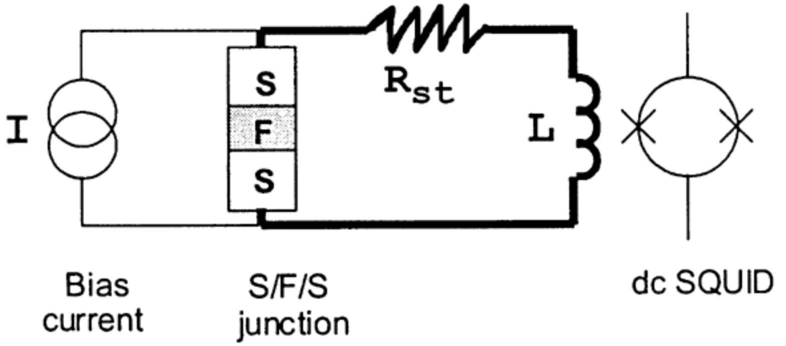


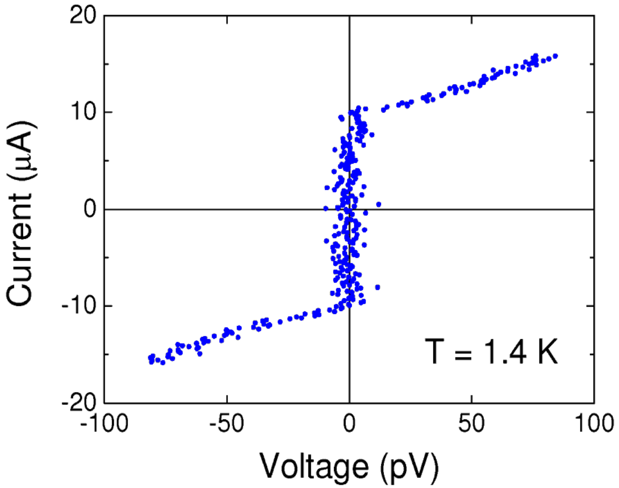
Spontaneously-broken symmetry

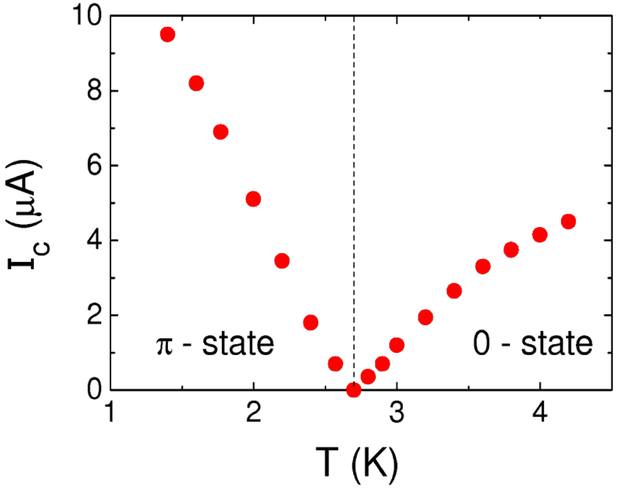
Spontaneous circulating current for $\beta_L > 1$
in zero applied magnetic flux

SFS Junctions --- mapping the 0-junction to π-junction transition

Current-Voltage measurement

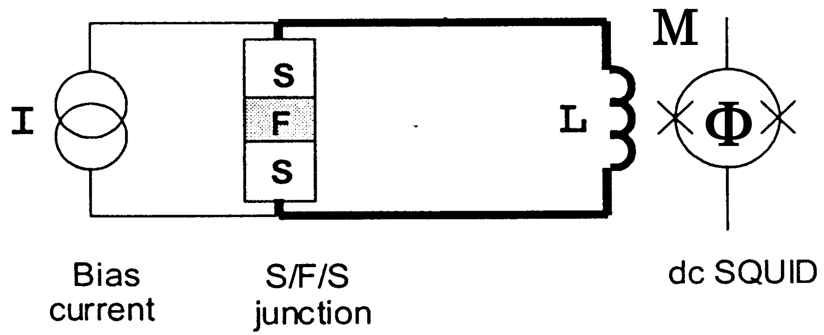


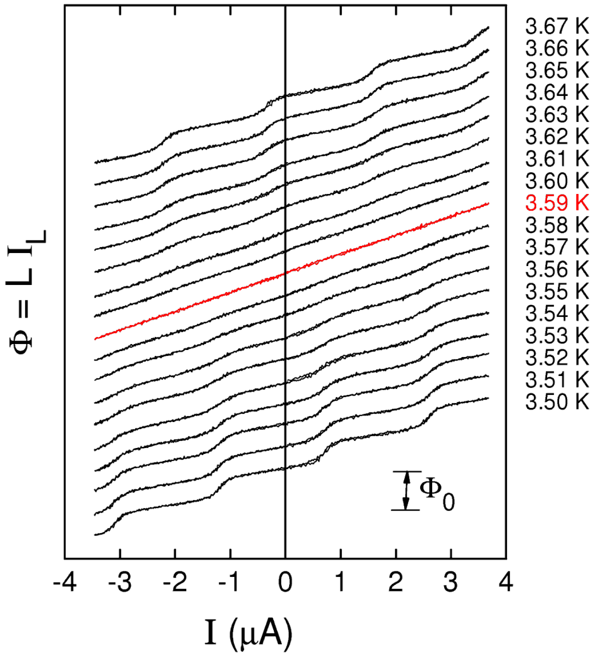


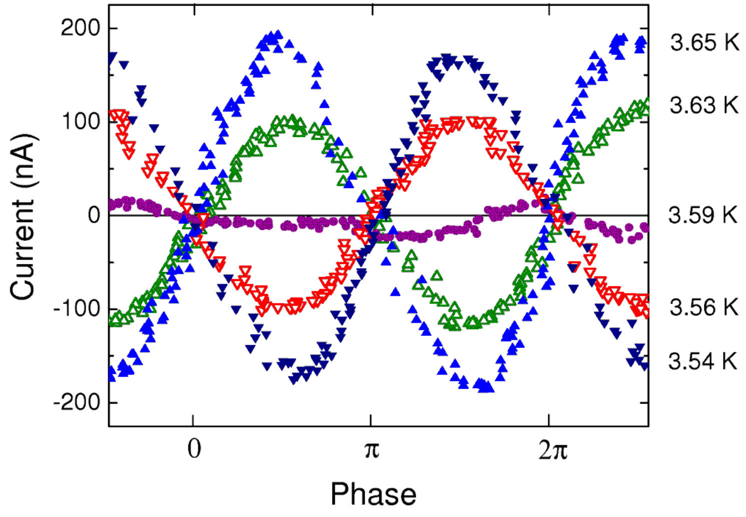


V. Ryazanov et al. (Chernogolovka)

Current-Phase Relation measurement





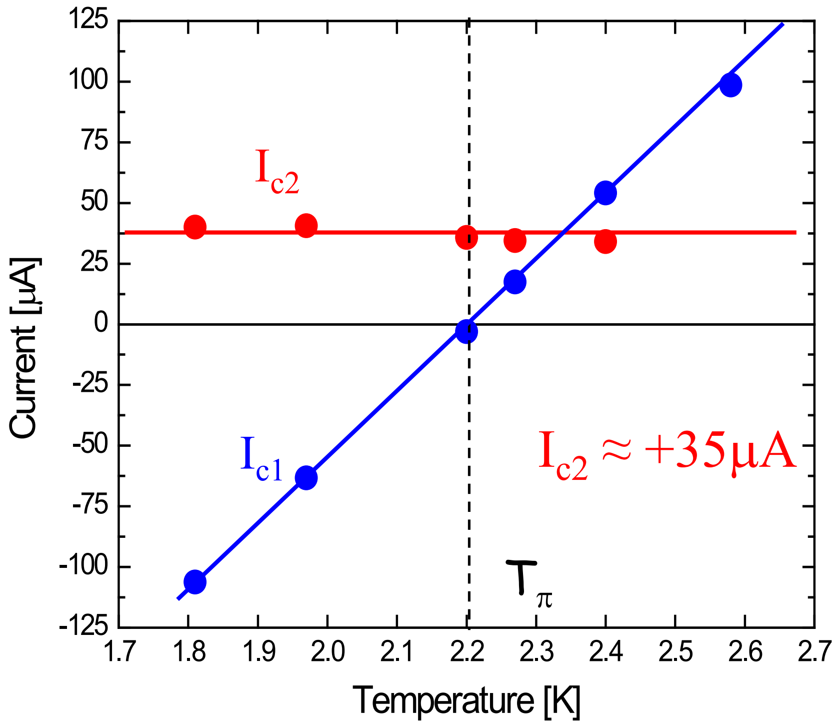


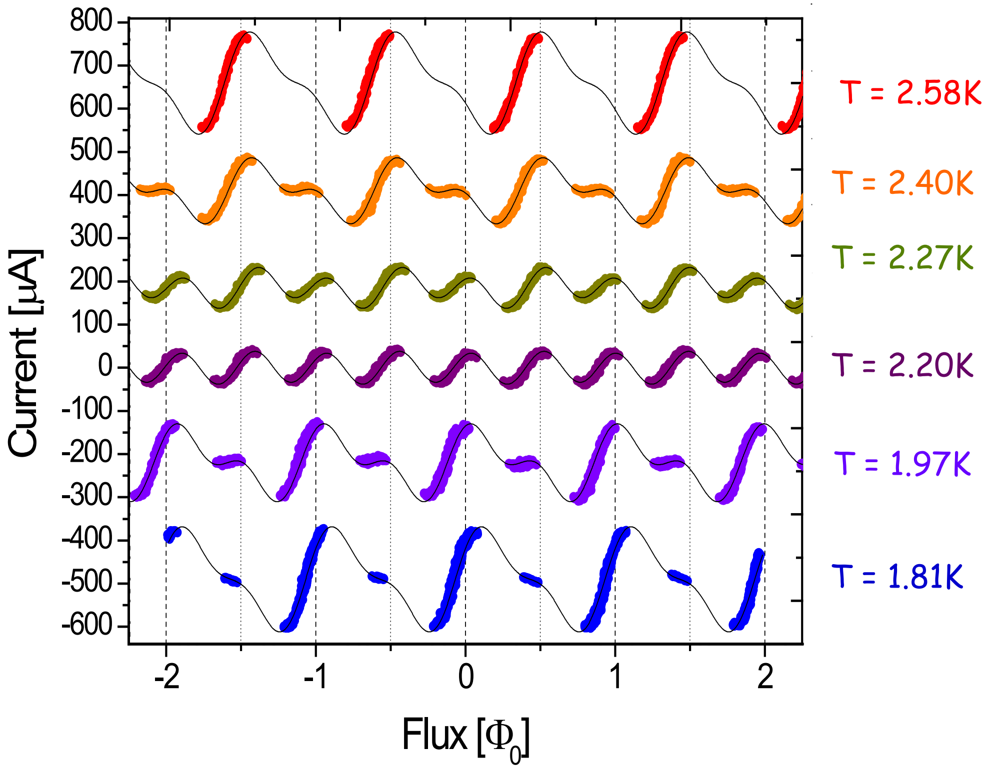
S. Frolov et al. (Urbana)

SFS Junctions --- looking for higher harmonics

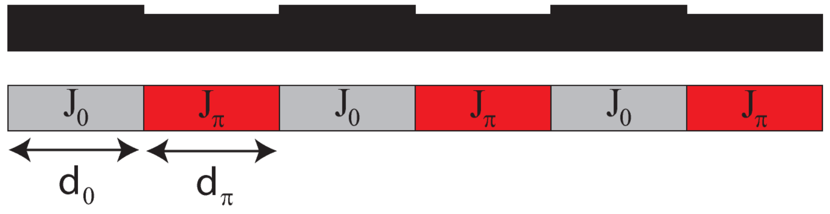
The vanishing of the first-order Josephson effect allows a search for higher harmonics in the CPR

$$I_s(\phi) = I_{c1} sin(\phi) + I_{c2} sin(2\phi)$$





Origin: rapid spatial changes in the critical current from 0-junction to π-junction



M. Stoutimore et al. (Urbana)

Josephson Interferometry: what it tells you

$$I_c(\Phi) = max \int_{-w/2}^{w/2} dy\, t\, J_c(y)\, cpr\left(\phi_0 + \phi_{op}(y) + \frac{2\pi}{\Phi_0}\left(\Phi + \int_0^{y} dy'\, d_m\, \delta B(y')\right)\right)$$

Critical current variation

Gap anisotropy

Domains

Charge traps

Current-phase relation

Non-sinusoidal terms

π-junctions

Exotic excitations e.g. Majorana fermions

Order parameter symmetry

Unconventional superconductivity

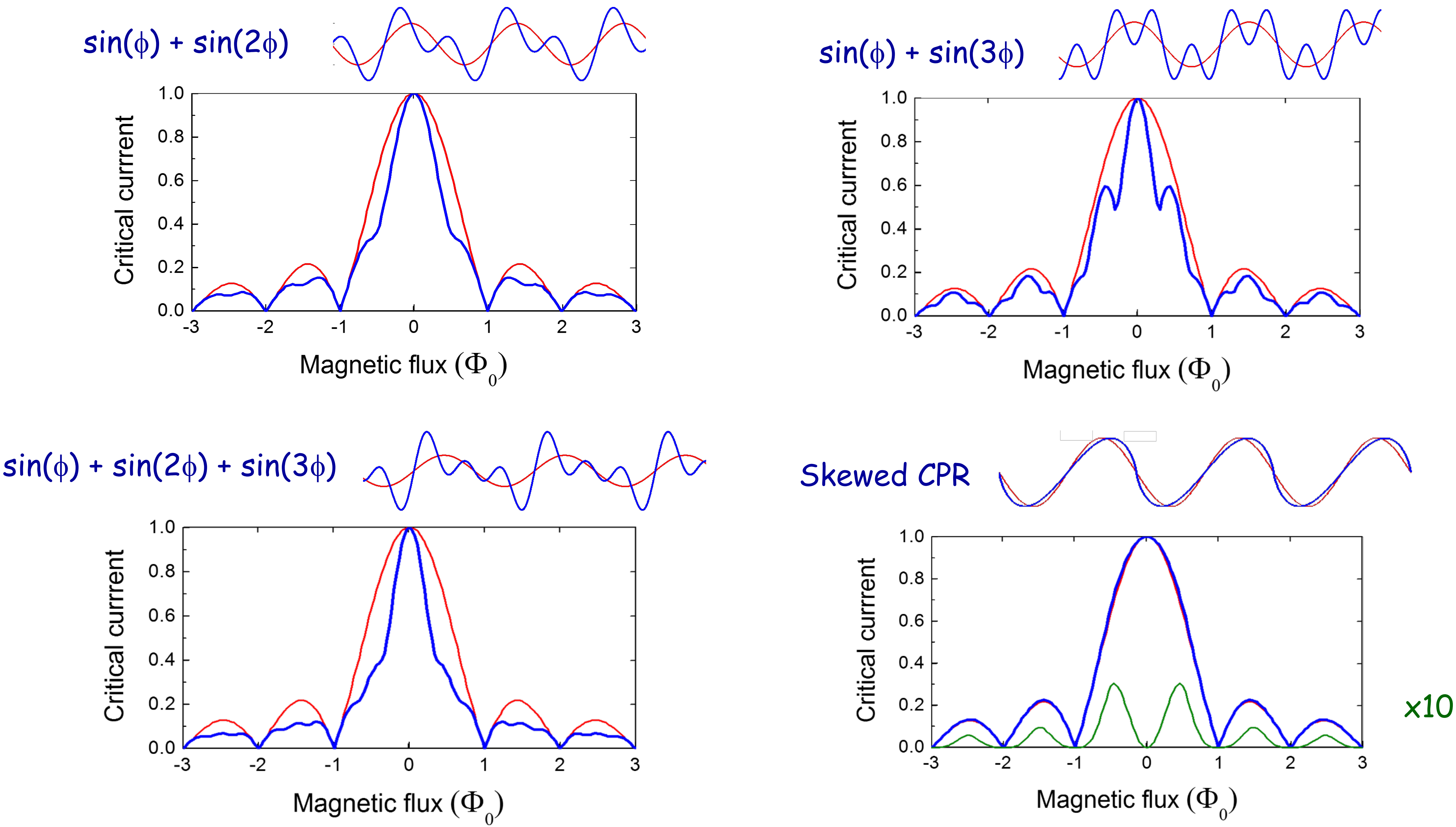
Magnetic field variations

Flux focusing

Trapped vortices

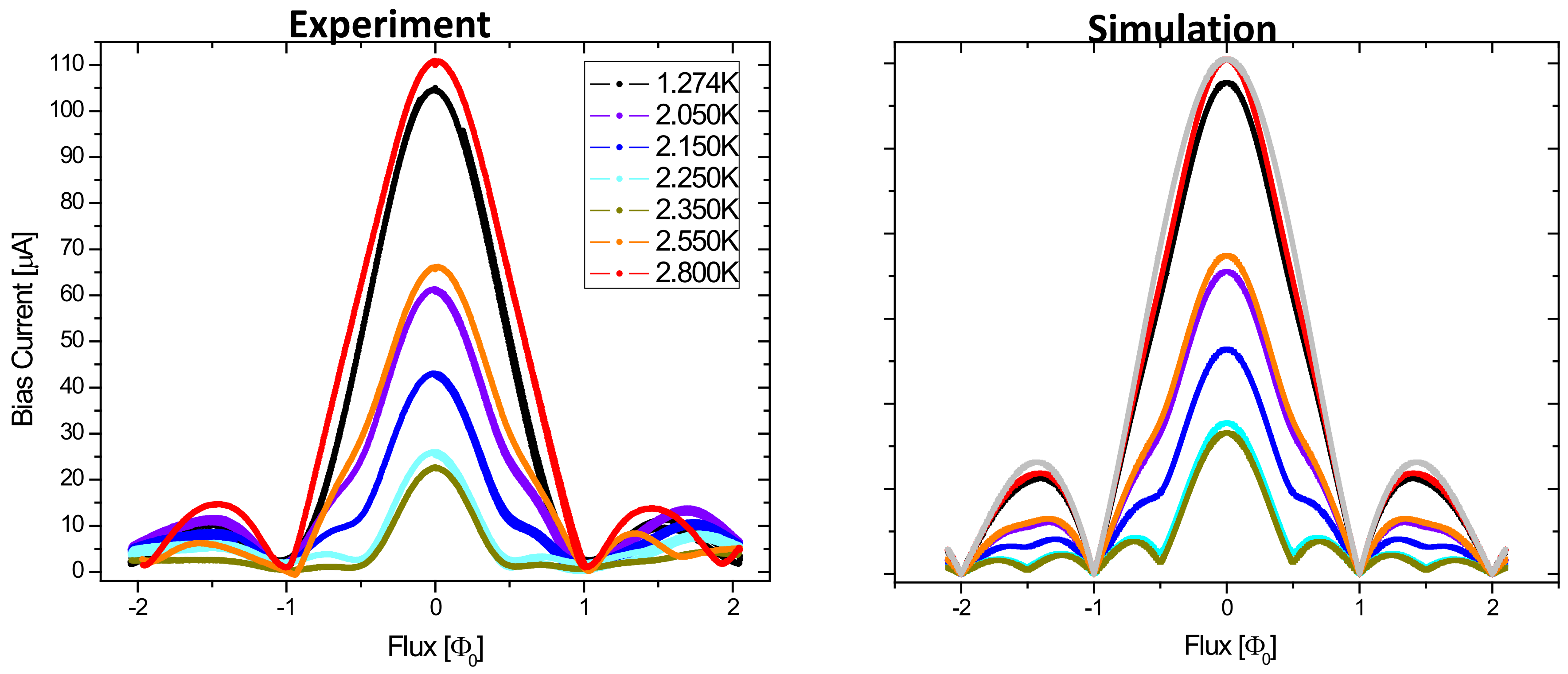
Magnetic particles

Effect of non-sinusoidal CPR on diffraction patterns



SFS: Josephson interferometry measurements

Nb-CuNi-Nb junctions

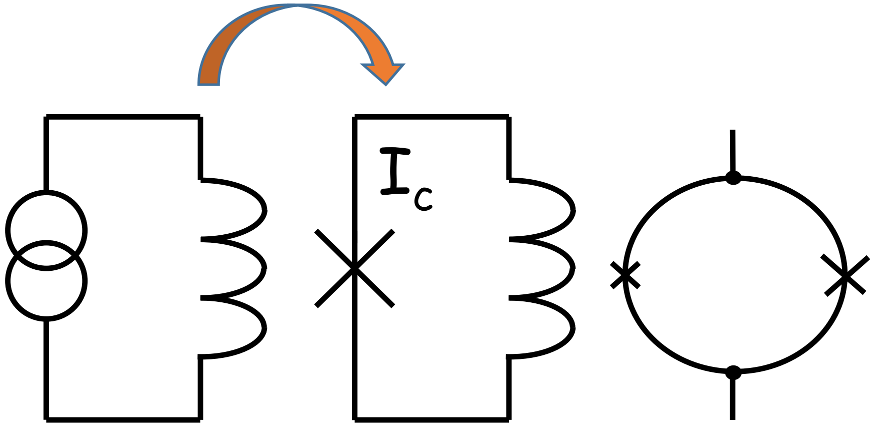


Simulation: $$I_s(\phi) = I_{c1} sin(\phi) + I_{c2} sin(2\phi)$$

Observe signatures of the non-sinusoidal CPR in the Josephson interferometry measurements

Direct measurement of the Current-Phase Relation

Screening technique (Jackel)

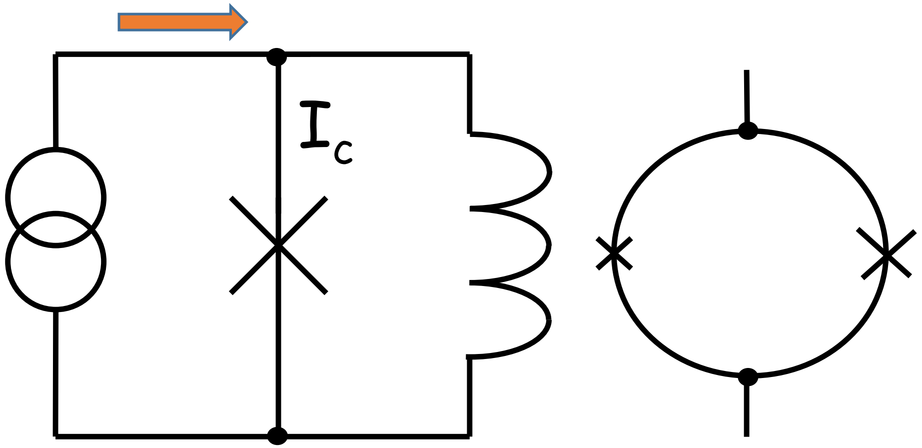


Junction embedded in SC loop (rf-SQUID)

Inject flux → induces circulating current

Detect flux with SQUID

Interferometer technique (Waldram)



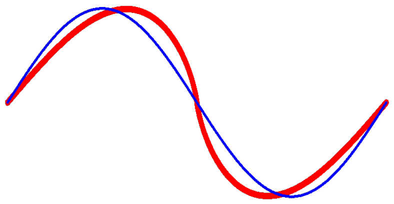
Junction in SC loop (rf-SQUID)

Inject current → divides according to phase

Detect flux with SQUID

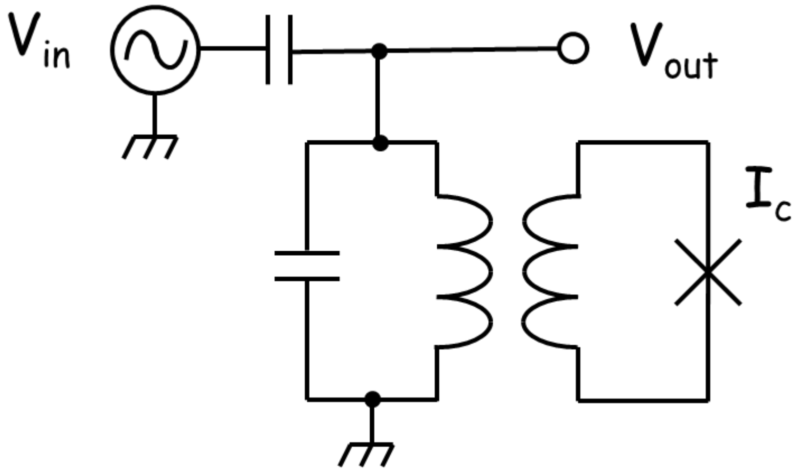
Extract CPR

First used to study the properties of superconducting microbridges → skewed CPR arising from an online inductance that gives extra phase



Direct measurement of the Current-Phase Relation

Dispersive technique (Silver, Deaver, Il'ichev)



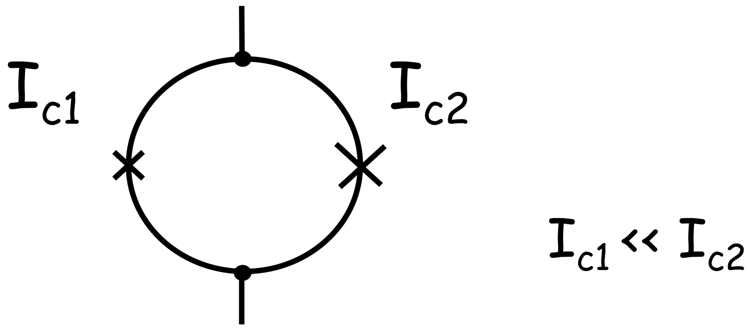
Junction embedded in SC loop (rf-SQUID) inductively-coupled to a tank circuit

Excite with rf signal → induces rf currents

Readout phase shift between V_{in} and V_{out}

Extract CPR

Asymmetric dc SQUID technique



Junction embedded in dc SQUID

Apply flux → induces circulating current

Measure critical current vs. flux

Modulation is dominated by the phase evolution of the small junction